

## Compound Interest Chapter 7

In order to find the amount of money in an investment account with interest rate  $r$ , principal  $P$ , time in years  $t$  and the number compounding periods  $n$  we need to use the

$$\text{formula } A = P \left( 1 + \frac{r}{n} \right)^{nt} .$$

Find the amount in the account using the compound interest formula given values for interest rate, principal, time and the number of compounding periods.

1)  $P = \$40,000$ ,  $r = 12\%$ ,  $t = 5$  years, compounded quarterly

2)  $P = \$140,000$ ,  $r = 10\%$ ,  $t = 7$  years, compounded monthly

3)  $P = \$30,000$ ,  $r = 7\%$ ,  $t = 10$  years, compounded quarterly

4)  $P = \$70,000$ ,  $r = 12\%$ ,  $t = 8$  years, compounded monthly

5)  $P = \$40,000$ ,  $r = 4\%$ ,  $t = 15$  years, compounded quarterly

6)  $P = \$100,000$ ,  $r = 2\%$ ,  $t = 5$  years, compounded monthly

## Discovering the number $e$

A mathematician by the name Jacob Bernoulli discovered the number  $e$  while working with this formula. What he did was to look at the situation where the amount invested is just one dollar, the interest rate is 100% and it is invested for one year. He left the number of times the interest

rate is compounded a variable. The formula becomes  $A = \left(1 + \frac{1}{n}\right)^n$ . By letting all of the other

variables equal to one he was able to study the affect that compounding has on the equation. If  $n$  is large what happens to  $A$ ? So he filled out a table similar to the one below. Think of yourself as a mathematician of the 17<sup>th</sup> Century and fill out the table below. Think about what you think will happen. Will the values on the right side of the table increase, decrease, converge to a specific value or grow to infinity?

$n$	$\left(1 + \frac{1}{n}\right)^n$
10	
100	
1000	
10000	
100000	
1000000	
10000000	
100000000	
1000000000	
10000000000	

On your calculator find the  $e$  button (same as division key). Push the blue shift button and then the blue  $e$  button. What do you get? We call this number  $e$  and it has many different kinds of

applications. So if we let  $n$  become infinitely large the original formula  $A = P\left(1 + \frac{r}{n}\right)^{nt}$  becomes

$A = Pe^{rt}$ . In the 1970's there was popular shampoo called pert. That is a helpful way to remember this formula.

Now re-do questions 1-6 but let the interest be compounded continuously which means there are an infinite number of compounding periods.

Find the number of years it will take for Joe to retire from worm harvesting given the amount he needs to be able to retire, the interest rate, the amount he currently has in his account and the number of compounding periods.

1)  $A = \$1000000$ ,  $r = 5\%$ ,  $P = \$40,000$  compounded quarterly

2)  $A = \$500000$ ,  $r = 7\%$ ,  $P = \$70,000$  compounded quarterly

3)  $A = \$2000000$ ,  $r = 11\%$ ,  $P = \$140,000$  compounded quarterly

4)  $A = \$1000000$ ,  $r = 12\%$ ,  $P = \$122,000$  compounded quarterly

5)  $A = \$1000000$ ,  $r = 5\%$ ,  $P = \$40,000$  compounded continuously

6)  $A = \$500000$ ,  $r = 7\%$ ,  $P = \$70,000$  compounded continuously

7)  $A = \$2000000$ ,  $r = 11\%$ ,  $P = \$140,000$  compounded continuously

8)  $A = \$1000000$ ,  $r = 12\%$ ,  $P = \$122,000$  compounded continuously